Zeliade Credit Analytics Library: Tranche Pricing Algorithm

November 2005

zeliade systems

Contact Information

Zeliade Systems
56 rue Jean-Jacques Rousseau
F-75001 Paris
France
Tel: +33 (0)1 40 26 17 29
Fax: +33 (0)1 40 26 17 81
Email: contact@zeliade.com
Web: http://www.zeliade.com/

Zeliade Credit Analytics Library: Tranche Pricing Algorithm
Zeliade Systems Technical Documentation, CDO series n. 2

TRADEMARKS

RELATED MATERIAL

Please contact Zeliade Systems to obtain the following related document:
Credit Index Calibration: How Do Models Perform ?, CDO Series, n. 1.

DISCLAIMER

Summary

A fast and reliable method for the computation of the (tranched) loss expectancy is needed to price bespoke deals and calibrate tranched indexes.

Monte Carlo algorithms are reliable, but not fast enough. The Quantization probabilistic method provides reference prices but is still not fast enough.

The fastest known way to compute tranche prices in the case of heterogeneous underlying portfolios is the Saddle-Point method. In its standard flavor, accuracy is not satisfactory.

Zeliade Tranche Pricing Algorithm enhances the Saddle-Point method at the algorithmic and numerical levels, yielding improved speed and accuracy.

I. INTRODUCTION

In order to price bespoke deals and calibrate tranched indexes, one needs a fast and reliable method for the computation of the (tranched) loss expectancy:

- Monte Carlo algorithms are reliable, but not fast enough.
- Quantization, a more elaborate probabilistic method that approximates a loss law at a given time with a prescribed accuracy is even more reliable but still not fast enough.
  However prices obtained by Quantization method can be safely regarded as reference prices.
- The fastest known way to compute tranche prices in the case of heterogeneous underlying portfolios is the Saddle-Point method.
  The accuracy of the classical Saddle-Point method is not satisfactory.
- The Zeliade Tranche Pricing Algorithm enhances the standard Saddle-Point method both at the algorithmic and numerical level yielding improved speed and accuracy.

Computation experiments timed in this document were performed on a mainstream laptop (AMD Athlon™ XP 2600 with 1.14Ghz processor and 512Mo RAM).

II. SPEED

In terms of speed, there is a sharp difference between the Quantization method and the Zeliade variant of the Saddle-Point method:

Table 1 – Speed results

Tranche		Running Time	
Lower	Upper	Zeliade enhanced Saddle Point	Quantization
0 %	3 %	0.14 sec.	9.03 sec.
3 %	6 %	0.14 sec.	9.06 sec.
6 %	9 %	0.14 sec.	9.03 sec.
9 %	12 %	0.12 sec.	9.03 sec.
12 %	22 %	0.12 sec.	9.06 sec.

III. CALIBRATION

The difference we have seen above implies signicant speed improvement for the calibration process:

Table 2 – Speed of the calibration process

	Zeliade enhanced Saddle Point	Quantization
DJ Itraxx	81.9 sec.	1314.1 sec.
DJ CDX	46.4 sec.	634.6 sec.

IV. ACCURACY

We present below comparative pricing outputs of both Saddle-Point and Quantization methods:

Table 3 – Accuracy (parameters calibrated on the DJ Itraxx Europe 5 Year, 20Jun-05)

Tranche		Model Mid		Difference
Lower	Upper	Zeliade enhanced Saddle Point	Quantization	
0 %	3 %	17.073 %	17.207 %	0.134
3 %	6 %	112.561 bp	112.558 bp	- 0.003
6 %	9 %	36.033 bp	35.981 bp	- 0.052
9 %	12 %	18.499 bp	18.695 bp	0.196
12 %	22 %	8.529 bp	8.458 bp	- 0.071

Table 4 – Accuracy (parameters calibrated on the DJ CDX.NA.IG 5 Year, 20-Jun-05)

Tranche		Model Mid		Difference
Lower	Upper	Zeliade enhanced Saddle Point	Quantization	
0 %	3 %	30.545 %	30.563 %	0.018
3 %	7 %	159.620 bp	159.438 bp	- 0.182
7 %	10 %	52.300 bp	52.465 bp	0.165
10 %	15 %	18.962 bp	18.687 bp	- 0.275
15 %	30 %	8.196 bp	8.170 bp	- 0.026

The good fit between the mids obtained through the two different techniques becomes apparent.

It is worth mentionning that Zeliade improvements to the SaddlePoint algorithm make it more accurate even in the case of Mezzanine tranches.

V. CONCLUSION

The Zeliade Tranche Pricing Algorithm, which relies on an enhanced Saddle-Point method, is both very accurate and exceptionally fast. This allows a full tranched index calibration within a minute on a standard laptop.

It also dramatically reduces the time needed for portfolio selection algorithms, hedge and VaR simulation or historical backtests which thus become feasible.

zeliade systems

56 rue Jean-Jacques Rousseau – F-75001 Paris – France
Tel: +33 (0)1 40 26 17 29 – Fax: +33 (0)1 40 26 17 81
Email: contact@zeliade.com
Web: http://www.zeliade.com/