Zeliade Systems

New Frontiers in Model Calibration

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Claude Martini, CEO of zeliade, describes the new trend in the art and science of model calibration.

Setting the stage: the current data paradox

The current state of the market data universe is paradoxical. The inexorable onward march of technology has made massive amounts of Trade and Quotes data available at a time scale below the miliseconds - even the ultimate granularity of every Trades and Quotes can be fed or downloaded at a reasonable cost from established providers. On another hand, the market itself generates new 'vanilla' product at a steady pace - the volatility markets (e.g. the VIX, its Futures and its Options) are a good example. This dream-of-a-statistician situation covers all the liquid universe of Futures and options on major equity and commodity indices.

On the other hand, a complex effect of a combination of liquidity and credit risk has led to a 'dislocation' of the interest markets with a blooming of Swap, FRA, Futures and XIBOR rates attached to every Tenor available. Among the multi-Tenor and maturities pageantry of quotes the issue is to select meaningful enough liquid ones, with very few guidelines. On Equity stock markets, contrary to Equity indices, option market data may be Trades or no-re-actualized Quotes, which make a careful selection altogether a challenge and a necessity. The central concept of consensual risk-free rate becomes an issue - the adequate discounting is tightly connected to funding and collateralization, which makes the whole process quite intricate.

Last but not least, access to market data has never been so laborious: the competition between the exchanges and between the major data market providers is pushing the market in a dangerous direction where you always need to pay more to get the same level of information or accuracy. Even worse, different sources may be inconsistent - as constated by the SEC when investing on the Flash Crash on May 6, 2010.

What are the consequences on the calibration protocol? Numerous. For instance:

- More configurations on "in" universe must be tackled - an Equity model might be calibrated on plain Vanillas, but also on a combination of the Vanillas, Variance Swaps and even other volatility instruments when available.
- The consistency of the various sub-components of the "in" universe has to be dealt with. For instance, Variance Swap quotes come from the OTC market and might show a discrepancy with the exchange-quoted universe of the volatility index. Some adjustment factor - or even, adjustment policy, naturally comes in.

The optimization conundrum

Once the "in" universe is selected, the best parameter of the model is calibrated via a suitable error minimization algorithm - minimizing the distance, in some relevant metrics, of the model to the market. This might be a purely historical (like often on Commodities) or purely static (that is to say, with market data of a given time stamp only) calibration. This stage, more than often, opens the Pandora box. Indeed:

- The model vanilla prices come from a well-designed and well-implemented piece of code - in the best case, even a well-unit-tested one. Yet the calibration will typically visit a very large region in the model parameters domain, and the pricing routine needs to produce the adequate figure in *all* cases. Typical issues arise when the vol-of-vol is too small in stochastic volatility models, or when a correlation coefficient is close to -1: the model will degenerate in these zones, and they deserve a special and cautious numerical treatment.

- The minimization procedure or algorithm depends on some 'purely numerical' parameters. Their settings are often tedious, and require a significant savoir-faire both at the market level and in the numerical space, and also require a solid testing infrastructure.

Despite its age, the academic research in the field of optimization is a very active field. A very nice achievment is the recent BOBYQA algorithm by Michael Powell which performs the optimization of a numerical function within a box, without knowledge of the derivatives. It outperforms all the previous algorithms with the same set of assumptions. BOBYQA grants that the function will *never* be evaluated ouside the box. Remarkably enough, many production implementation of calibration algorithms use box-constrained optimization algorithms that may require during iteration the evaluation of the function outside the box - which can mean, pricing a Call with a negative volatility - the behavior of such implementations is nothing short of a miracle. Few implementations of optimization algorithms do grant the strict in-box property - some care is needed!

Simple ideas to disambiguate calibration

The recent years have seen the emergence of non-optimizer dependent calibration procedures. This is closely connected to the design of data-driven parsimonious models - those models have been crafted, in fact, with the goal to avoid, if possible, the dependency on 'heavy' optimization algorithms.

The SVI model is emblematic of this family. Put forward in 2004 by Jim Gatheral, it displays a remarkable ability to calibrate almost any volatility smile (except, may be, the shortest maturity one). Yet, and despite the fact that it consists of an explicit elementary close formula with 5 parameters, it was difficult to calibrate these 5 parameters in a stable an non-ambiguous way. It turns out that a simple *reparameterization* of the model allows to largely disambiguate the process, by reducing it to a constrained linear regression with an explicit solution available, followed by a box minimization in dimension 2. This has been developed in [2] and implemented in Zeliade's ZQF.

Altough a very simple remark, the ability to get a stable calibration was a big step forward, and this quasi-explicit calibration of the SVI model has a great success and is used world-wide by actors who face the calibration challenge.

Excatly the same idea can be applied to other contexts, like the calibration of Variance Swap curves in parametric models like the Heston or (still Jim Gatheral) the Double Lognormal model. The corresponding calibration codes are short, robust, fast, and depend at most of a one dimensional minimization algorithm like the very efficient Brent algorithm.

Of course there is a trade-off at some stage: it is not very surprising to design easy-to-use calibration procedures for these models, since they were designed for that purpose. On the drawback side, they are not always first class citizens in the realm of aribtrage-free models in mathematical finance. It might happen that the SVI parameterization can not, *theoretically*, correspond to an arbitrage-free volatility smile.

A parallel move, much more demanding, is the quest for explicit formula for existing mainstream models, typically in short term asymptotics, or long term, or extreme strikes asymptotics. Antoine Jacquier (Imperial College) and his co-authors is one of the pioneer in this area. Even if the techniques involved may be very refined ones, the output is eventually a close formula which can be used, for instance, to infer a first guess of the model parameter by a clever combination of different results. In this respect, the differential geometry approach pioneered by Henry-Labordère ([7]) is a milestone for small-time asymptotics.

Vanna Volga and friends

The Vanna-Volga (VV) methodology has become a standard in the Forex landscape. It is a perfect illustration of a data-driven approach designed with the goal of taking the smile into account at the cheapest cost. There again, one of its more appealing feature is the bypassing of 'heavy' optimization steps - the calibration amounts to the well-defined inversion of a 3x3 matrix.

The evil comes back when the VV method is applied to price exotics like barriers or Touch options. The VV coefficients need to be adjusted, or weighted, to account for the correct behavior at the barrier. The First-Exit-Time or Survival weightings are the most commonly used among practitioners, yet the way to get a final weight out of them is not completely settled. In this respect, the beautiful idea of Kurt Smith (Curtin Univeristy, [3]) deserves a special mention: just compute the FET first, then apply the standard VV to a correctly extrapolated input smile at the fake maturity given by the FET.

Karasinski ([1], 2005) has observed that the VV idea can be applied with another model than the Black-Scholes model with the ATM volatility. In fact, it could be applied to any not-perfeclty-calibrating model, as a second stage ajdustment to perfeclty fit the market. This is a deep line of thougth, which is likely to become a standard post-calibration tuning in the near future.

A 20 years old problem solved

In the context of the pricing of structured products, the usual paradigm is to apply a calibration procedure to a set of market data which any reasonable model should reprice correctly, and then to price the complex product on the same undelyers with the calibrated model. This is especially meaningful when some of the input items of the calibration will serve as hedging instruments.

Unfortunately, it is well known, well studied and well documented that a bunch of models can almost match the input market data and yield eventually drastically different prices for the structured product at hand (The famous: A perfect calibration, now what? of [4]). This has even led to a sound definition of model risk of an exotic product - in the different but related context of Uncertain Volatility Model, Avellaneda and his co-authors had paved the way with the Lagrangian UVM model and the Weighted Monte Carlo algorithm. Rama Cont, and recently Patrick Hénaff ([5]), have designed model risk frameworks along those lines.

Given this multi-model state of fact, a key challenge is to *compute* the range of all possible prices among the calibrating models. This was pending for 20 years and has just been solved by Galichon, Labordère, Touzi([6]). Their approach relies on an optimal transport interpretation of the transition probabilities between 2 maturities mixed with the dual formulation of Monge-Kantorovitch - one eventually gets a constrained stochastic control problem which can be effectively solved. This work has been sponsored by the Chaire Financial Risks sponsored by Société Générale - a great interaction of academia and practicioners.

Of course, in practice, the most important point is to feed the algorithm with pertaining constraints - liquid Variance Swap quotes, for instance, will give a lot of information about the jump component of the underlying model, and drive the bounds towards regions of practical interests. Regarding multi-asset products, this is where the shoe pinches. Despite recent attempts like Adil Reghai's local correlation, or the academic production on the Wishart approach to multi dimensional stochastic volatility, there is no clear breakthrough in this area. Remarkably enough - given the relatively cold reception in its early days of the UVM model, the UVM paradigm applied to correlation uncertainty seems to be widely used, at least in dimension less than 6.

New tools in the bag

The increase in computing power has provoked a paradigm shift in many calibration-related issues. People are more willing to use easy-to-implement yet long-running algorithms than hyper fast yet highly tuned, over engineered and hyper sensitive ones. In hybrid calibration algorithms, the use of Bayesian methods like the Markov Chain Monte Carlo methods becomes widespread.

In a fast changing environment, adaptive estimation techniques become more and more essential. Machine learning (ML) algorithms, and especially Support Vector Machines, are now a standard tool in the technology toolbox of Quant/IT desks. ML algorithms are used in various areas for calibration purposes. The most well-known is algo trading - from statistical arbitrage to high frequency trading - where well-trained machine learning algorithms are second to none as adaptive forecasters. The come into play in historical satistical estimation, where several recent works suggest that they outperform classical estimator based on the maximum likelihood to estimate models in the ARCH/GARCH family.

Last but not least, the Cloud has become a standard weapon in a technology arsenal. Public Cloud portails like Windows Azure or the fantastic Picloud in the Python world are mostly used in R&D departments or academic labs, whereas *private* Cloud infrastructures may be more suitable for production needs. The Cloud is the right answer to a highly varying computing power need. If your needs are more regular, with less peaky shapes, GPU computing may be a better solution - even if they remain by several order of magnitudes more difficult to exploit than the Cloud, software providers start to make use of GPU cards transparently for their low-level layers.

A very hot new trend at the intersection of high power computing and machine learning is the "News analytics": exploiting the stream of internet feeds like Twitter or others to do forecasting, detect new trends or changes in model regimes. High power computing is needed when the answer should come quickly, as in the case of high frequency trading.

Conclusion

The art and science of model calibration quickly evolves with new markets and data environments, academic innovations, and the availability of massive computing power. The forthcoming years will certainly see a blooming of new calibration methodologies with a growing role of Machine Learning techniques, with new challenges for analytics software developers - and also for academia.

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